4. Load

- The torque developed by the motor is given by (Newton's 2nd Law)

$$T_{el} = T_l + J\frac{d\omega_m}{dt}$$

where

- T_{el} is the torque developed by the motor
- J is inertia seen by the motor's shaft
- ω_m is the motor's speed
- T_l is load torque

- The load torque consists of
  - Friction torque, T_F
  - Windage torque T_W
  - Torque required doing useful mechanical work, T_L
- The load torque is given by:

$$T_l = T_F + T_W + T_L$$
$$T_F = B\omega_m$$
$$T_W = C\omega_m{}^2$$

Load torque-speed characteristics

- $T_L = k$ or constant
  - Examples:
    - Machines work like shaping, cutting, grinding or shearing
    - Cranes during the hoisting
    - Conveyors
- Constant, $T_L = k\ \omega_m$
  - Examples:
    - Separately excited DC generator connected to a constant resistor

- $T_L = k\,\omega_m^{\,2}$
  - Examples:
    - Fans
    - Pumps
    - Compressors
    - Ship propellers
- Constant, $T_L = k/\omega_m$
  - Examples:
    - Lathes
    - Boring machines
    - Milling machines
    - Steel mill coiler

- $T_L = k_0 + k_1\omega_m$
  - Examples:
    - Hoist
    - Elevator
- Constant, $T_L = k_0 + k_1\omega_m + k_2\,\omega_m^2$
  - Examples:
    - Compressor

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Inertia

- The motor is usually connected to the load through a set of gears
- The gears have teeth ratio and can be treated as a torque transformer
- The gears are used to amplify the torque on the load side that is at lower speed compared to the motor's speed
- The gears can be modeled from the following facts:
  - The power handle by the gear is the same on both sides
  - Speed on each side is inversely proportional to the tooth number

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \tau$$

$$T_1\omega_1 = T_2\omega_2 \Rightarrow \frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{1}{\tau}$$

where

- N_1 and N_2 are the teeth numbers in the gear
- τ is gearbox ratio
- ω_1 is the motor's speed
- ω_2 is the load speed
- T_2 is the load torque
- T_1 is the load torque referred to the motor's shaft

- The load inertia reflected to the motor shaft can be calculated as:

$$\frac{1}{2}J_{L,ref}\,\omega_1^{\,2} = \frac{1}{2}J_L\,\omega_2^{\,2}$$

$$J_{L,ref} = J_L\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{J_L}{\tau^2}$$

where

- J_L is the load inertia
- $J_{L,ref}$ is the load inertia reflected to the motor's shaft

- The total inertia seen by the motor is given by:

$$J = J_m + J_{L,ref} = J_m + \frac{J_L}{\tau^2}$$

where

  - J_m is the motor's inertia for the shaft

- Maximum power transfer occurs if the load inertia reflected to the motor shaft is made to match the motor inertia. In this case, maximum acceleration of the load will result

$$T_{el} - T_l = J\frac{d\omega_1}{dt} \Rightarrow T_{el} - T_l = \left(J_m + \frac{J_L}{\tau^2}\right)\tau\frac{d\omega_2}{dt}$$

$$T_{el} - T_l = \left(J_m\tau + \frac{J_L}{\tau}\right)\alpha_2 \Rightarrow \frac{T_{el} - T_l}{\alpha_2} = F(\tau) = J_m\tau + \frac{J_L}{\tau}$$

$$\frac{\partial F(\tau)}{\partial\tau} = 0 \Rightarrow J_m = \frac{J_L}{\tau^2} = J_{L,ref}$$